The following are outlines for the three linear algebra lectures for the Integration Workshop. There are many textbooks which are useful resources for this material, but the following I find especially useful:


1. **Vector Spaces and Linear Transformations**

1.1. **Vector Spaces.** Definitions of vector space, subspace, quotient space, linear independence, direct sum, basis, coordinates, and dimension.

1.2. **Linear Transformations.** Definitions of linear transformation, and its image and kernel. Correspondence between linear transformations and matrices, null space and column space. Gaussian elimination, reduced row echelon form, and rank. The Rank-Nullity Theorem. Solving linear systems.

1.3. **Dual Spaces.** Definition of the dual space $V^*$ of a vector space $V$, and the dual basis. If $V$ is finite dimensional, $V$ is non-canonically isomorphic to $V^*$, and $V$ is canonically isomorphic to $V^{**}$.

2. **Eigenvalues and Jordan Form**

2.1. **Determinant and Eigenvalues.** Definition and computation of determinant, eigenvalues, and eigenvectors. Change of basis and diagonalization. Characteristic and minimal polynomials.

2.2. **Jordan Form.** Definition of semisimple and nilpotent. Jordan form of a matrix and endomorphism.

3. **Bilinear Forms**

3.1. **Inner Product Spaces.** Inner products, orthogonal and orthonormal vectors, Gram-Schmidt process to find an orthonormal basis.

3.2. **Bilinear Forms.** Definition of bilinear form and non-degeneracy. Symmetric, alternating, and Hermitian forms. Isometries of forms, normal operators.

3.3. **Classification.** Classification of forms over $\mathbb{R}$ and $\mathbb{C}$. Every bilinear form can be uniquely written as the sum of a symmetric and alternating form. Change of basis for a bilinear form, and diagonalization of forms over $\mathbb{R}$ and $\mathbb{C}$.